Date: TBA Time: TBA	Class: 09 Prepared by: Md Imanul Huq
Duration: 1.5H	

Today's Topics Theme: RELATION

Today Topics to be Covered:

1. What is Relation? 2. The Language of Relations 3. Relations on Sets 4. Inverse of a Relation	5. Reflexivity Symmetry and Transitivity 6. Equivalence Relation 7. Equivalence Class

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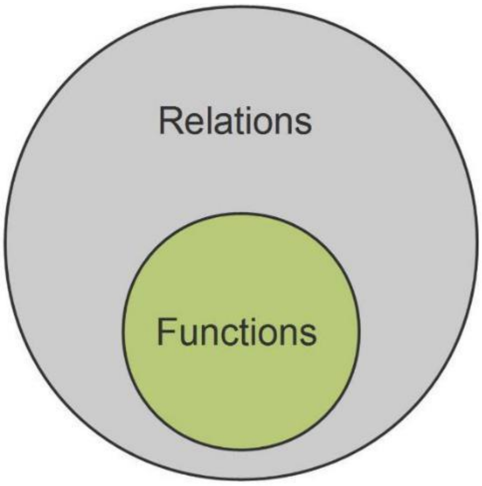
1. Relation
   - Some relationships make sense and others don't
   - Functions are relationships that make sense
   - All functions are relations, but not all relations are functions



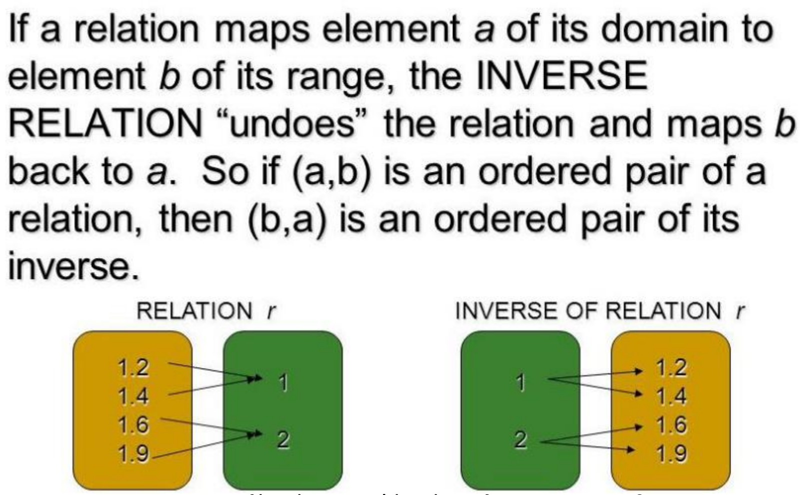
2. The Language of Relations
   - What odes relationship means in terms of mathematics
   - We can relate two objects in different way
   - For example, if a number is bigger or not
   - Let A={1,2,3} B={2,3,4}

     1 R 2 Valid
     1 R 3 Valid
     1 R 4 Valid
     2 R 2 Not Valid
     2 R 3 Valid
     2 R 4 Valid
     3 R 2 Not Valid
     3 R 3 Not Valid
     3 R 4 Valid
     {(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)} based on rule R
   - A = {2,3} B = {2,3,4}
     R=(x-y)/2 ϵ Z
     2 R 2 Valid
     2 R 3 Not Valid
     2 R 4 Valid
     3 R 2 Not Valid
     3 R 3 Valid
     3 R 4 Not Valid
     {(2,2), (2,4), (3,3)}
3. Relations on Sets
   - A= {1,2,3} B = {1,3,5,6}
     S means that x<y = {(1,3), (1,5), (1,6), (2,3), (2,5), (2,6), (3,5), (3,6)}
     Now AXB={(1,1),(1,3),(1,5), (1,6), (2,1), (2,3),(2,5), (2,6), (3,1), (3,3), (3,5), (2,6) } S ⊂ AxB

Let A and B be sets. A relation R from A to B is a subset of A x B. Given an ordered pair are (x, y) in AxB, x is related to y by R, written, x R y, if, and only if, (x, y) is in R
The set A is called the domain of R and the set B is called its co-domain.

- Example 01: Define a relation C from R to R for any (x,y) ϵ RxR where (x,y) ϵ C iif x^2+y^2= 1
  Is (1,0) ϵ C? Y/N
  Is (0,0) ϵ C? Y/N
- Example 01: X = {1,2,3} p(x) = {{}, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3}}
  For all the A and B ϵp(x)
  A R B ⇔ A has the same number of elements as B

4. Inverse of a Relation



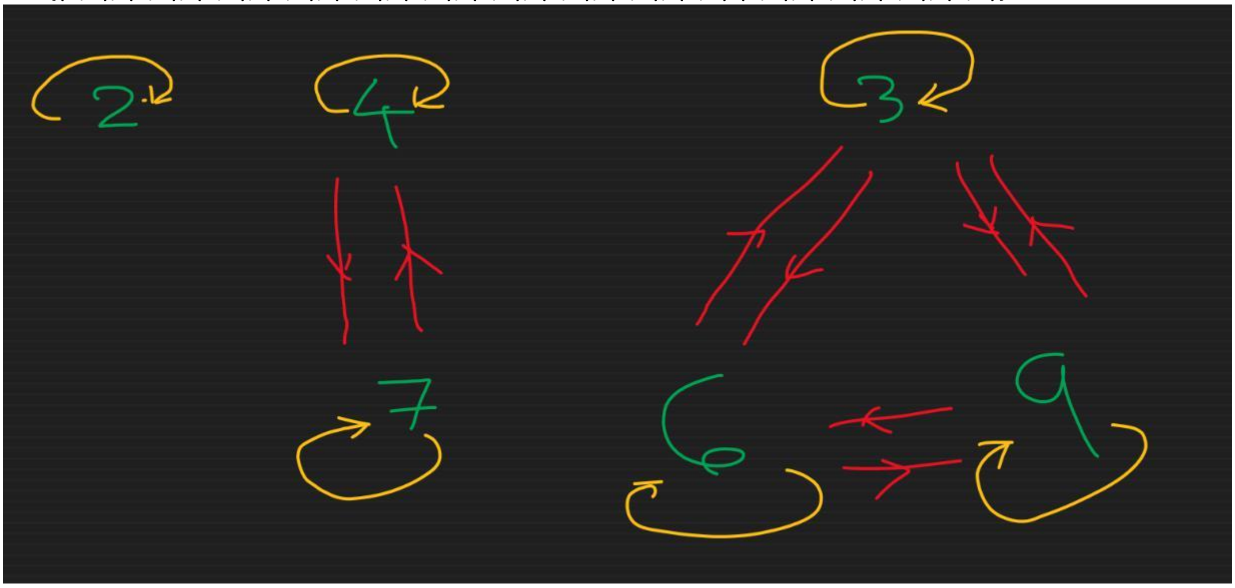
Source:

https://slideplayer.com/slide/7493180/

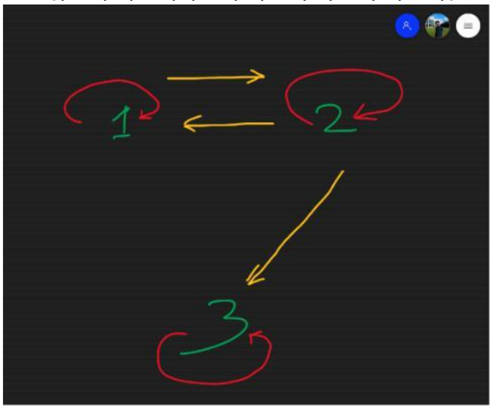
- NOTE: R_{-1} = {(y,x) ϵ BxA|(x,y) ϵ R} is inverse of
  R = {(x,y) ϵ AxB|(x,y) ϵ R}
- Example 01: A = {2,3,4}, B = {5, 2, 8} x R y ⇔ x/y
  R = {(2,6), (2,8), (3,6), (4,8)}
  R^{-1} = {(6,2), (8,2), (6,3), (8,4)}

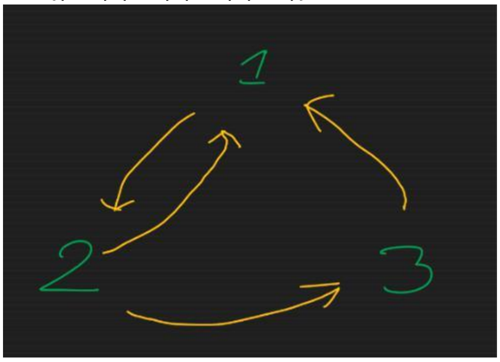
05. Reflexivity Symmetry and Transitivity

- A = {2,3,4,5,7,9} R: x, y ϵA x
  R y ⟷ 3/(x-y)
  R = {(2,2), (3,3), (4,4), (6,6), (7,7), (9,9), (6,3), (3,6), (9,6), (9,6) (3,9), (9,3), (4,7), (7,4)}



I. R is Reflexive ⟷ for all xϵA, (x,x)ϵR
II. R is Symmetric ⟷ for all x,yϵA if (x,y)ϵ then (y,x)ϵR
III. R is Transitive ⟷ for all x,y,zϵA if (x,y)ϵR and (y,z)ϵR then (x,z)ϵR

6. Example of Reflexivity Symmetry and Transitivity
   - Example: A = {1,2,3}
     R = {(1,1), (2,2),(3,3), (1,2), (2,3), (2,1)}
     

   - Example: A = {1,2,3,4}
     R = {(1,2),(2,1),(2,3),(1,3)}
     

   - Example: {(1,2)}
     NOTE: This is Transitive

7. Equivalence Relation

An equivalence relation on a set X is a subset of $X \times X$, i.e., a collection R of ordered pairs of elements of X, satisfying certain properties. Write "$x\, R\, y$" to mean (x, y) is an element of R, and we say "x is related to y," then the properties are

1. Reflexive: $a\, R\, a$ for all $a \in X$,
2. Symmetric: $a\, R\, b$ implies $b\, R\, a$ for all $a,\ b \in X$
3. Transitive: $a\, R\, b$ and $b\, R\, c$ imply $a\, R\, c$ for all $a,\ b,\ c \in X$,

where these three properties are completely independent. Other notations are often used to indicate a relation, e.g., $a \equiv b$ or $a \sim b$.

Source: https://www.wolframalpha.com/input/?i=equivalence+relation

08. Equivalence Class

An equivalence class is defined as a subset of the form $\{x \in X : x\, R\, a\}$, where a is an element of X and the notation "$x\, R\, y$" is used to mean that there is an equivalence relation between x and y. It can be shown that any two equivalence classes are either equal or disjoint, hence the collection of equivalence classes forms a partition of X. For all $a,\ b \in X$, we have $a\, R\, b$ iff a and b belong to the same equivalence class.
A set of class representatives is a subset of X which contains exactly one element from each equivalence class.
Source: https://mathworld.wolfram.com/EquivalenceClass.html